Edge Chain Detection by Applying Helmholtz Principle on Gradient Magnitude Map

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***Abstract*—In this paper, we present an efficient edge chain detection algorithm by applying the Helmholtz principle on the gradient magnitude map of an image. An edge chain validation method is proposed which uses the "relative number of false alarms" (RNFA) instead of the traditional "number of false alarms" (NFA). The edge chains are detected first and then validated according to their RNFA values. In this way, edge chains that are weak in gradients but meaningful in vision can be detected. To evaluate the proposed edge chain detector in quantity, an edge chain detection benchmark which consists of 25 labeled images in different scenes was built. The proposed edge chain detector was tested in this benchmark, and the experimental results sufficiently demonstrate that the proposed edge chain detector outperforms the state-of-the-art methods.**

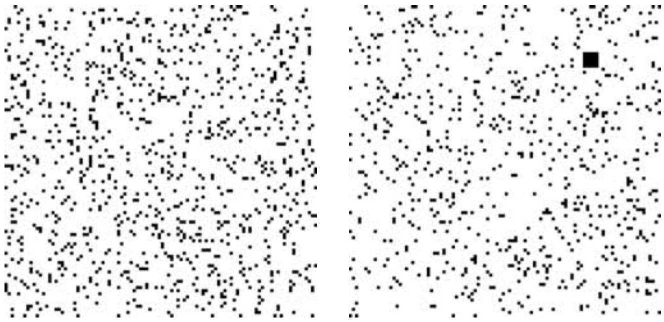
1. Introduction

Geometric structure detection on an image is an important and classical problem in image processing and computer vision, which has been studied for decades. Edge chains are one of the most widely used geometric structures which can be used to represent the silhouettes of an image. As a low level information of an image, edge chains can be applied in line segment detection [1], [2], object recognition [3], image segmentation [4], and so on.

Based on the understanding: "edge is most often defined as an abrupt change in some low-level image feature such as brightness or color" [5], traditional edge detectors usually take two steps to extract edge segments: feature image extraction and feature image thresholding. Numerous edge detectors have been proposed in the past decades [6]–[9] based on this idea. The Canny operator [6] is a widely used edge detector which finds the peak gradient magnitudes orthogonal to the edge directions by applying a non-maximum suppression. However, it uses the gradient magnitudes as information, which makes it difficult to distinguish the faint edge pixels from the noise. Wang et al. [10] proposes the "supporting range" to distinguish those weak edge pixels from their surroundings and applied a segment-based hysteresis thresholding approach to verify the edge segments. Edge Drawing is a recently proposed edge detector: "computes a set of anchor edge points in an image and then links these anchor points by drawing edges between them" [11]. Edge Drawing is fast and uses more direction information than Canny on its novel edge linking process. In the work of [12], the use of the Helmholtz principle gives a new view on both boundary and edge chain detections. However, this work mainly focuses on the geometric event: a strong contrast along a level line of an image, thus to some extent it can not be considered as a proper edge detector. "Conversely, the detection algorithm provides a check tool to accept or reject edges proposed by any other algorithm" [12]. EDPF [13] develops the original work of Edge Drawing into a parameter-free edge detector by applying the Helmholtz principle on the validation check of the detected edge chains.

The Helmholtz principle is popularly applied in the detection of image structures like line segments [1], [14], [15], edges and boundaries [12], [13], continuous curves [16] and vanishing points [17]. The Helmholtz principle does not use an a priori or learned model, but applies the *a contrario* uniform random assumption. The *a contrario* assumption is based on a certain background model. For example, in the line segment detection [14], the background model is the image's gradient orientation map, and the assumption is that the gradient orientations of pixels are independent and uniformly distributed in the range $(-\pi, \pi]$ on the gradient orientation map. The LSD detector [14] first applies a region growing method to obtain a line-support region, and then a rectangle is fitted as an approximation of the region, finally the Helmholtz principle is applied to validate the meaningfulness of this region by calculating the "number of false alarms" (NFA) of this region according to the number of aligned orientations in it. In the edge and boundary detection [12], the background model is the level lines of an image, and the assumption is that the contrast (gradient magnitude) at a point on any level is mutually independent. For a piece of level line $\mathtt{E}$, the minimum contrast u of pixels on $\mathtt{E}$ is searched and the meaningfulness of the event "each pixel of $\mathtt{E}$ has a contrast larger than u" is calculated via the Helmholtz principle.

Despite that the Helmholtz principle is well studied and applied in both the orientation map and the level lines, its application on the gradient magnitude map is still not well discussed yet. The main reason is that the value of N_{conf} [18], which is one of the key factors to calculate the NFA and denotes the number of different possible configurations one could have for the searched image structure such as edge chain or line segment, is hard to be determined for the application like the edge chain detection on a background model of the gradient magnitude map. In this contribution, we firstly give a new view on applying the Helmholtz principle on the gradient magnitude map, and then propose an edge chain detection algorithm which uses the "relative number of false alarms" (RNFA) instead of the traditional "number of false alarms" (NFA) to get rid of false alarms. To evaluate the proposed edge chain detector in quantity, an edge chain detection benchmark



(a) randomly distributed (b) a square structure

Figure 1. Example images with the size of 100×100 pixels: (a) 200 independent black pixels randomly distributed on the image; (b) an image with a 5×5 black square structure.

was built which consists of 25 labeled images captured from different scenes.

2. Helmholtz Principle on Gradient Map

2.1. Helmholtz Principle

Figure 1 shows two simulated images, in Figure 1(a) there is a image with 100×100 pixels, among which 200 independent black pixels are randomly distributed. In the image shown in Figure 1(b), there is a 5×5 black square structure. Comparing to the image shown in Figure 1(a), we will sense that such a square structure shown in Figure 1(b) could not be arose just by chance. But how to measure this sense in quantity? The computational Gestalt theory and Helmholtz principle [12], [18] give a systematic solution. Before we have a deep look in the Helmholtz principle on the detection of image structures, some basic concepts are introduced as follows:

- *event*: a geometric structure on an image, for example the black square shown in Figure 1(b).
- *object*: the basic element to form an event, for example the pixels in the black square shown in Figure 1(b).
- *quality*: a common character that is shared by all the objects of an event, for example the quality that all the pixels in the black square shown in Figure 1(b) are all "in black".

In the Helmholtz principle, the sense "a structure could not arise just by chance" is defined as the expectation of the number of occurrences of this structure (event) under the *a contrario* uniform random assumption, which is also known as the "number of false alarms" (NFA). According to the Helmholtz principle, an event is meaningful if the NFA of this event is very small. The NFA is formulated as follows:

$$\mathrm{NFA} = N_{\mathrm{conf}} \times \mathcal{B}(n, k, p), \tag{1}$$

where N_{conf} denotes the number of different possible configurations one could have for the searched event, which means that there probably are N_{conf} events in theory on the image; p represents the probability that a given object has a considered quality; and $\mathcal{B}(n, k, p)$ is the tail of the binomial distribution which means the probability that at least k objects out of the observed n ones have this quality under the independence assumption.

As a summary, there are three key factors in the Helmholtz principle: (1) the perspective meaningful event; (2) the theoretical number N_{conf} of the event on the image; (3) the probability p of the considered quality. Take the two images in Figure 1 for example, in Figure 1(a) there is no perspective meaningful structure (event) observed, while in Figure 1(b) the black square is sensed as a meaningful structure. Assume that we have already obtained the black square on the image shown in Figure 1(b) by some detection methods, the rest of the problem is how to calculate the N_{conf} and $\mathcal{B}(n, k, p)$. In the case of the black square, the event now is "a square made up of black pixels", the object is "pixel" and the quality is "pixel in black". The definition of an event gives the estimation of N_{conf}, as a square is determined by two diagonal vertexes, each vertex can be any pixel on the image, thus $N_{\mathrm{conf}} = (100 \times 100)^2$. The probability of a 5×5 black square is $\mathcal{B}(25, 25, \frac{1}{5}) = (\frac{1}{5})^{25}$, thus the NFA of the black square is $(100 \times 100)^2 \times (\frac{1}{5})^{25} = 3.3 \times 10^{-10}$, which is a really small value and means that this square can hardly occur in a background model where the black pixels are randomly and independently distributed with a probability of $1/5$, so according to the Helmholtz principle the 5×5 black square on the image shown in Figure 1(b) is perspective meaningful.

2.2. Helmholtz Principle on Level Lines

To apply the Helmholtz principle on edge chain detection, the first problem we encounter is the definition of an edge chain event, because an edge chain can be anywhere with any shape and any length, which makes it difficult to be expressed in a certain model. To solve this problem, in the work of [12] the "level line" was introduced, and an edge chain is defined as "a piece of level line along which the contrast of the image is strong", so the N_{conf} of the edge chain event is formulated as:

$$N_{\mathrm{conf}} = \sum_i l_i(l_i - 1)/2, \tag{2}$$

where l_i is the pixel number of the i-th level line. The considered quality now is "each pixel of a level line $\mathtt{E}$ has a contrast equal or greater than u", and the probability of this quality is formulated as:

$$H(u) = \frac{1}{M}\#\{\mathbf{x} \in \mathbf{I} | g(\mathbf{x}) \geq u\}, \tag{3}$$

where $\mathbf{I}$ is the image, $g(\mathbf{x})$ denotes the contrast (gradient magnitude) of a pixel $\mathbf{x}$ and M is the number of pixels whose gradient magnitudes are not equal to zero on the image, i.e., $M = \#\{\mathbf{x} \in \mathbf{I} | g(\mathbf{x}) \neq 0\}$. So to apply the Helmholtz principle on the edge chain detection based on the level lines, first of all we should get the level lines of the image, then for an edge chain with l pixels, the smallest gradient magnitude u on this chain is found, and finally the NFA of this edge chain is defined as $N_{\mathrm{conf}} \times H(u)^l$.

We can see that applying the Helmholtz principle on the edge chain detection is not a straight forward work because the level lines should be obtained in advance. In the work of EDPF [13], the level lines are replaced with edge chains obtained by the Edge Drawing method for convenience without convincing proofs. In fact, both the level lines based and the edge chains based N_{conf}s are just approximations of the exact value of N_{conf}, and it is still difficult to find a convincing method to calculate the value of N_{conf} for the edge chain event.

2.3. Helmholtz Principle on Gradient Map

Let $\mathbf{I}$ be $w \times h$ image and $\mathbf{G}$ be the integral gradient map of $\mathbf{I}$ by applying a gradient operator on $\mathbf{I}$. In our entire work the 3×3 Sobel operator is applied, and the gradient magnitude $g(\mathbf{p})$ of a pixel $\mathbf{p}$ in $\mathbf{I}$ is calculated as follows:

$$g(\mathbf{p}) = \sqrt{(g_x(\mathbf{p}))^2 + (g_y(\mathbf{p}))^2}, \tag{4}$$

where $g_x(\mathbf{p})$ and $g_y(\mathbf{p})$ represent the gradients of the pixel $\mathbf{p}$ in $\mathbf{I}$ in the horizontal and vertical directions, respectively.

For each integral gradient magnitude level $u \in [1, g_{\max}]$ where $g_{\max}$ is the maximum gradient magnitude level in $\mathbf{G}$, the number of pixels whose gradient magnitude level is equal or greater than u is denoted as $k(u)$, thus the probability of the considered quality that "a pixel on $\mathbf{I}$ whose gradient magnitude level is equal or greater than u" is defined as:

$$P(u) = k(u)/M, \tag{5}$$

where $M = w \times h$ is the size of $\mathbf{I}$. This definition is similar to that of Eq. (3) in form, but different in one of the basic conception of the Helmholtz principle. As we have stated before, the quality is a common character that is shared by all the objects of an event, and the probability of the quality represents the distribution of the background model. By setting M as the number of pixels whose gradient magnitudes are greater than zero on the image, Eq. (3) implies that the background model is the level lines of an image, while Eq. (5) means that the objects with this quality is distributed randomly on the whole gradient map.

Definition of NFA - *Number of False Alarms*. Given an event $\mathcal{E}$ (a detected structure) made up of l pixels on an image, N_{conf} is the theoretical number of $\mathcal{E}$ on the image, u is the minimal gradient magnitude of these pixels, the NFA of $\mathcal{E}$ on the gradient magnitude map is defined as:

$$\text{NFA} = N_{\text{conf}} \times P(u)^l. \tag{6}$$

In some applications, it is difficult to give a good approximation of the N_{conf}, for example the value of N_{conf} for edge chain is hard to obtained as we have discussed in Section 2.2. In this case, we proposed to use the "relative number of false alarms" (RNFA) to validate edge chains.

Definition of RNFA - *Relative Number of False Alarms*. Given an event $\mathcal{E}$ (a detected structure) whose binomial probability is $\mathcal{B}(n,k,p)$, and $\mathcal{E}_r$ is a minimal meaningful event (MME) whose binomial probability is $\mathcal{B}(n_r,k_r,p_r)$. The relative number of false alarms of $\mathcal{E}$ to $\mathcal{E}_r$ is defined as:

$$\text{RNFA} = \frac{N_{\text{conf}} \times \mathcal{B}(n,k,p)}{N_{\text{conf}} \times \mathcal{B}(n_r,k_r,p_r)} = \frac{\mathcal{B}(n,k,p)}{\mathcal{B}(n_r,k_r,p_r)}, \tag{7}$$

where N_{conf} is the number of different possible configurations one could have for the searched event, and we simply say that the event is meaningful than the minimal meaningful event (MME) if RNFA < 1. As we can see from Eq. (7) that, all configurations of a given type of event on the image share the same value of N_{conf}, which means that the exact value of N_{conf} can be eliminated if a reference case can be found. In this way, the problem of finding a good approximation of N_{conf} is converted into the searching for the minimal meaningful event (MME), which can be very simple in some cases.

3. Edge Chain Detection

In many cases, it is difficult to find a MME reference case, but in the application of the edge chain detection, it works. The basic idea is that "a meaningful line segment on the image is also a meaningful edge chain". So, given a $w\times h$ image $\mathbf{I}$, first of all we should get the minimum length L_{mm} of a meaningful line segment, which can be very well solved according to the works of LSD and CannyLines [14], [15]:

$$L_{mm} = -2.5\log(M)/\log(p), \tag{8}$$

where $M = w \times h$ is the size of $\mathbf{I}$ and $p = 1/8$. Thus, we can give the definition of the "minimal meaningful edge chain event" of an image:

Definition of MME_{edge} - *Minimal Meaningful Edge Chain Event*. A minimal meaningful edge chain event is defined as the edge chain with a size of L_{mm} and a minimal gradient magnitude equals to $g_{\min}$.

The $g_{\min}$ is a user defined parameter which is set constant, in Section 4.2.1 we will demonstrate how to find the best value of $g_{\min}$ for all the applications. Thus the RNFA of an edge chain can be reformed as follows:

$$\text{RNFA}_{\text{edge}} = \frac{\mathcal{B}(n,k,p)}{P(g_{\min})^{L_{mm}}}. \tag{9}$$

If $\text{RNFA}_{\text{edge}} < 1$, we simply say that the edge chain is meaningful.

3.1. Edge Chain Detector

An edge chain should have the following qualities on a gradient magnitude map: (1) made up of edge pixels (zero-crossing pixels [6]); (2) smooth orientation deviations between consecutive edge pixels. Based on this observation, an efficient edge chain detector is proposed as follows:

(1) First, given a gray image $\mathbf{I}$, the gradient magnitude map $\mathbf{G}$ and gradient orientation map $\mathbf{O}$ of $\mathbf{I}$ are calculated by applying a certain gradient operator (a 3×3 Sobel was applied in our work).

(2) Then, the non-maximum suppression procedure is applied on $\mathbf{G}$, the gradient magnitudes of those suppressed pixels are set zero and the remaining ones are edge pixels, the set of which is denoted as $\mathbf{E}$.

(3) Third, the set $\mathbf{E}$ is sorted in descending order according to the gradient magnitudes. The foremost unprocessed edge pixel in $\mathbf{E}$ is selected as the initial seed pixel $\mathbf{p}_{seed}$. The 8-neighborhood of the $\mathbf{p}_{seed}$ is searched. If there exists a 8-neighbor who is an edge pixel and the orientation deviation between it and the seed pixel is less than θ, we consider this pixel to be the next seed pixel and added it to the current edge chain. The seed growing of the current edge chain is conducted iteratively until all the pixels in this chain is processed, and then we begin with another edge chain from the rest of $\mathbf{E}$.

(4) Each edge chain detected in the step (3) is validated by the Helmholtz principle on edge chain proposed in Section 2.3 to get rid of the false alarms.

It's worth noting that there are two internal parameters in the proposed edge chain detector: θ and $g_{\min}$. The value of θ

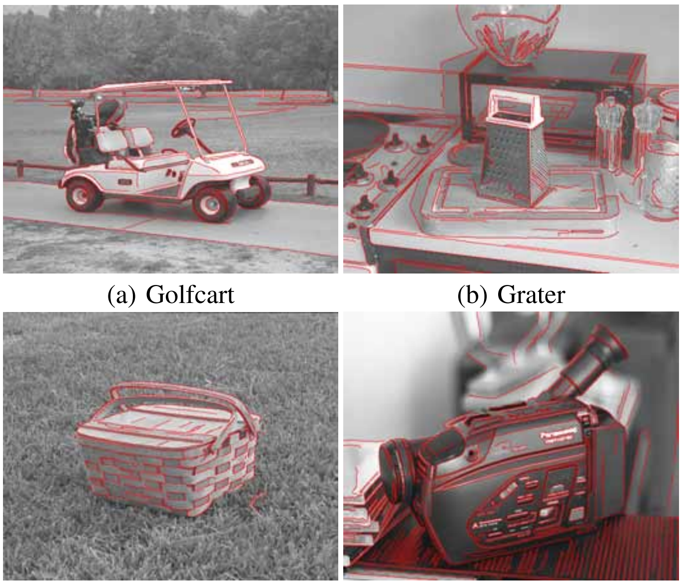
is set as $\pi/4$ for constant, so the performance of the proposed algorithm can be adjusted by setting a customized $g_{\min}$, and the bigger the value of $g_{\min}$ is the more meaningful the final edge chains are. In Section 4.2.1 we will demonstrate how to find the best value of $g_{\min}$ for all the applications.

4. Experimental Results

4.1. Edge Chain Benchmark

To evaluate the performance of the proposed edge chain detector, we built a benchmark with ground truth edge chains labeled in a semi-automatic way. The reason why we don't use two widely used public databases: the BSDS dataset and the RUG database is that both of these two databases are more or less labeled based on objects instead of edge chains.

There are 25 images in our benchmark, which are semi-automatically labeled by edge pixels collection with manually selected seed pixels, most of which were selected from the EDC dataset [1] [19] despite of several natural images that are too difficult for human to label. The images cover a range of textured and non-textured, man-made and natural scenes. Figure 2 shows four representative images and the corresponding labeled edge chains.



(a) Golfcart (b) Grater

(c) Picnic Basket (d) Videocamera

Figure 2. Four representative images and the corresponding ground truth edge chains on the proposed benchmark.

To evaluate the accuracy of the edge chain detection result, we use the same F-score metric as EDPF [13]. Let DC be the set of edge pixels detected by a certain method, GT denotes that of the ground truth data, the precision (P) and recall ratio (R) are defined as follows:

$$P = \frac{\#\{\mathrm{DC} \cap \mathrm{GT}\}}{\#\{\mathrm{GT}\}} \quad \text{and} \quad R = \frac{\#\{\mathrm{DC} \cap \mathrm{GT}\}}{\#\{\mathrm{DC}\}}. \tag{10}$$

The F-score is defined as $F = 2PR/(P+R)$.

1. Available at http://marathon.csee.usf.edu/edge/edge_detection.html

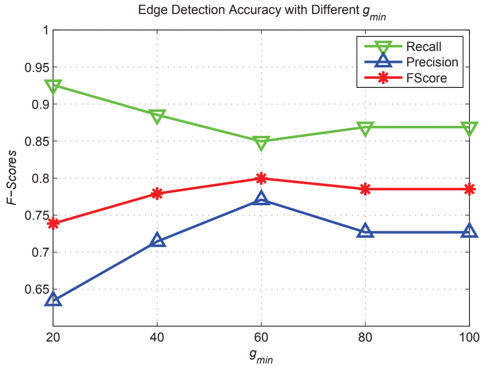


Figure 3. Different performances of our proposed edge chain detection algorithm with different values of $g_{\min}$.

4.2. Evaluation on Edge Chain Detector

4.2.1. Choice of Best $g_{\min}$. According to the definition of $\mathrm{MME}_{\mathrm{edge}}$, $g_{\min}$ is the minimum gradient magnitude which makes an edge chain with a size of L_{mm} meaningful on an image. According to the work of Wang et al. [10], the gradient magnitude of a pixel along with the "supporting range" of this pixel determines its saliency. In our work, we only take the gradient magnitude into consideration, and assume that an edge chain is meaningful in vision if its gradient magnitude is stronger than a certain threshold. To find out the best value of $g_{\min}$, we set $g_{\min} = 20, 40, 60, 80, 100$, respectively. Figure 3 shows the edge chain detection results on the benchmark with different values of $g_{\min}$, we can see that $g_{\min} = 60$ produces the greatest detection precision, which leads to the highest F-score. So in our works, we set $g_{\min} = 60$ for constant. However, as we have stated in Section 3.1 that the performance of the proposed algorithm can be adjusted by setting a customized $g_{\min}$, and the bigger the value of $g_{\min}$ is the more meaningful the final edge chains are.

4.2.2. Comparison of Level Lines, Edge Chain and RNFA. As we have mentioned in Section 2.2 that in the works of [12] and EDPF [13], the level lines and the edge chains are used to calculate the value of N_{conf}, respectively. In this section, we will compare the performance of these two methods with our proposed RNFA method on the benchmark we built. The level lines were created with the level quantization step equal to 2, the edge chains were detected by the method proposed in Section 3.1, and the value of $g_{\min}$ was set as 60. Table 1 shows the average accuracies of these three methods on all the 25 images in the benchmark. From Table 1, we can see that the proposed RNFA method achieved the best scores on precision and the F-score, which are close to the level lines based method and better than those of the edge chain based method. In fact the values of N_{conf} of the four images in Figure 2 calculated based on the level lines and edge chains are 565950545, 402641165, 555485560, 414409382 and 785282, 961799, 469320, 1255510, respectively. In average, the values of N_{conf} calculated based on level lines are around 800 times those calculated based on the edge chains, which is the reason why the edge chain based method gains the higher recall ratios but lower precisions than the other two methods. As a conclusion, the proposed RNFA method can achieve close or

TABLE 1. COMPARISON BETWEEN LEVEL LINES, EDGE CHAIN AND RNFA.

Methods	Level Lines			Edge Chain			RNFA		
Measurements	R	P	F	R	P	F	R	P	F
Average	0.87	0.73	0.78	**0.89**	0.66	0.75	0.84	**0.77**	**0.80**

even better accuracy as the level lines based method, however we don't have to obtain the level lines of an image in advance.

4.2.3. Comparison with State-of-the-Art Methods. To sufficiently evaluate the performance of our proposed RNFA based edge chain detection method, we compared it with other four state-of-the-art edge detection methods, including: EDPF [13], ED [11], SREdge [10] and CannyPF [15]. The source codes of ED and EDPF can be obtained from the Edge Drawing library [20], the source code of our previously proposed CannyPF is publicly available [2] and the source code of SREdge was implemented by us according to the paper. Table 2 shows the average accuracies of these algorithms on all the 25 images in the benchmark. From Table 2, we can see that the proposed RNFA method achieves the highest values on both precision and F-score, which is much better than the EDPF on the second place. The edge chain based method SREdge also performs very well considering the fact that it applies the saliency instead of the Helmholtz principle to validate the edge chains. We can also find out that the algorithms EDPF and RNFA that apply the Helmholtz principle as a validation procedure, achieve higher precisions than those of CannyPF, ED and SREdge that do not apply the Helmholtz principle, which proves the effectiveness of the Helmholtz principle. Figure 4 shows the edge detection results of these five algorithms on six test images in the benchmark. We can observe from Figure 4 that the proposed RNFA method achieves the best performance, EDPF and RNFA generate less false alarms than the other three methods, which is consistent with the conclusion drawn from Table 2.

5. Conclusion

In this work we developed the Helmholtz principle on the gradient magnitude map of an image based on probability theory, and proposed a new conception named "relative number of false alarms" (RNFA) as a supplement of the "number of false alarms (NFA) which is a key conception in the Helmholtz principle. We apply the RNFA on the gradient magnitude map for structure detection from image, and proposed an efficient and useful edge chain detector. To evaluate the proposed edge chain detector in quantity, an edge detection benchmark was built which consists of 25 labeled images captured from different scenes. Experimental results show that the proposed RNFA based edge detection method achieves the highest F-score comparing with four state-of-the-art edge detection methods.

Acknowledgment

This work was partially supported by the National Natural Science Foundation of China (Project No. 41571436), the National Natural Science Foundation of China under Grant 91438203, the Hubei Province Science and Technology Support Program, China (Project No. 2015BAA027), the Jiangsu Province Science and Technology Support Program, China (Project No. BE2014866), and the South Wisdom Valley Innovative Research Team Program.

2. Available at http://cvrs.whu.edu.cn/projects/cannyLines/

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TABLE 2. COMPARISON OF RNFA, EDPF, SREDGE, ED AND CANNYPF.

Methods	RNFA			EDPF			SREdge			ED			CannyPF		
Measurements	R	P	F	R	P	F	R	P	F	R	P	F	R	P	F
Average	0.85	0.77	**0.80**	0.81	0.65	**0.71**	0.79	0.61	**0.65**	0.83	0.55	**0.62**	0.88	0.49	**0.59**

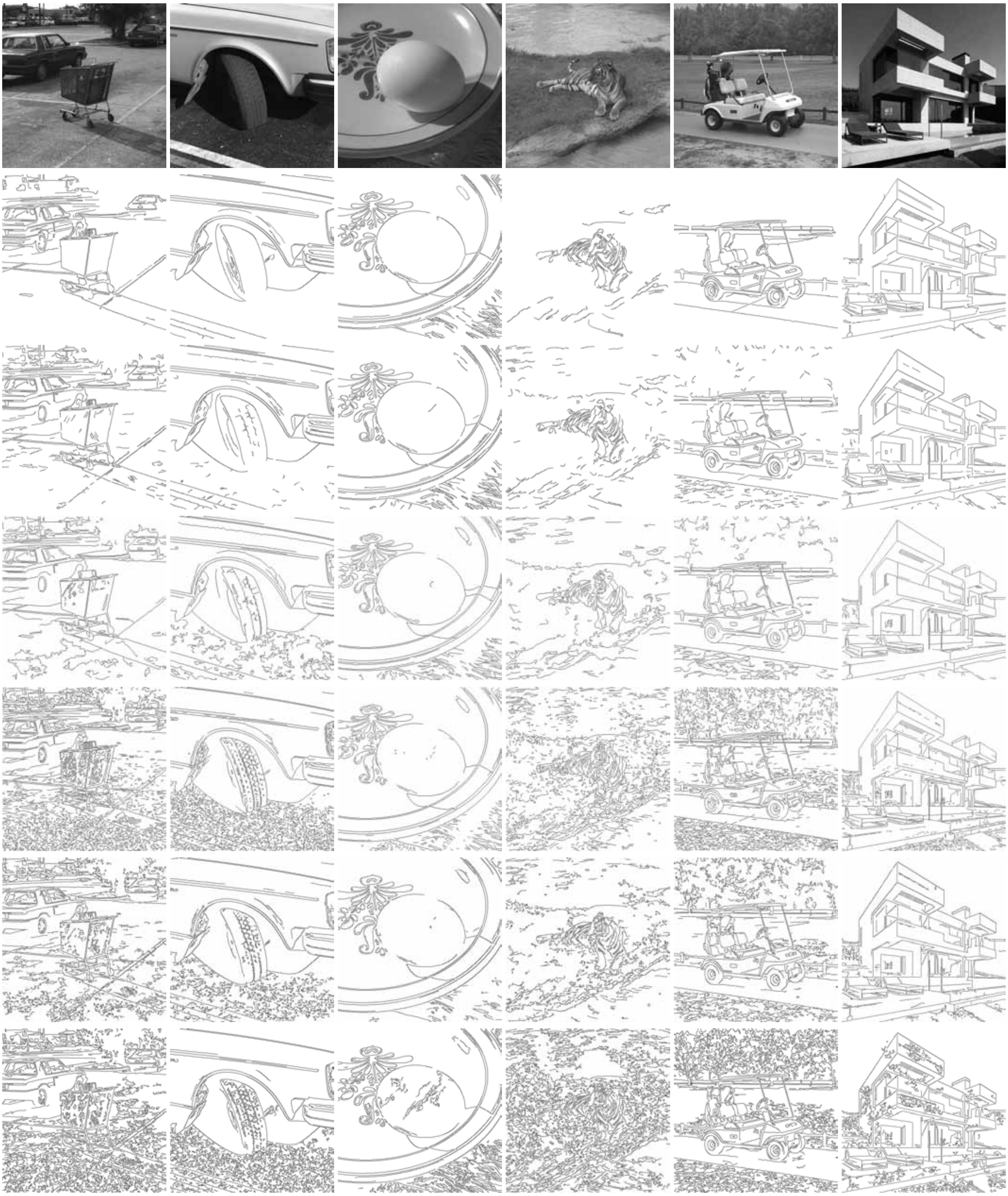

Figure 4. Six test images in our benchmark in the first row, ground truth edge chains in the second row, and edge chains detected by RNFA, EDPF, ED, SREdge and CannyPF from the third row to the final row, respectively.